USING THE KALMAN FILTER IN TESTS OF CONVERGENCE

Les Oxley, Department of Economics, University of Waikato

David Greasley, Department of Economic History, University of Edinburgh

Abstract. Tests of the convergence hypothesis or the tendency for per capita income levels to narrow over time, have generally utilised cross-sectional data and resulted in conflicting evidence. Recently, Bernard and Durlauf (1995), have proposed time series tests and using these methods Greasley and Oxley (1995), find evidence of convergence in GDP per capita in the case of Australia and the UK.

However, Hall, Robertson and Wickens (1992), and recently St.Aubyn (1996), have proposed a Kalman-filter-based test of convergence which overcomes some of the problems raised in Greasley and Oxley (1998). In this paper we utilise the St.Aubyn approach considering data on GDP per capita and wages per capita for several OECD countries. Evidence is presented in favour of the existence of separate 'convergence clubs' including a European and Nordic group.

1.0 Introduction

Tests of the convergence hypothesis, or the tendency for per capita income levels to equalize over time, have attracted considerable attention, see for example, Barro and Sala i Martin (1992), and Dowrick and Nguyen (1989). For an excellent overview of the area including economic underpinnings and econometric evidence see Barro and Sala i Martin (1995). In a series of recent developments Bernard and Durlauf have extended the traditional testing methodology by arguing in favour of time series rather than cross-sectional based tests, see Bernard and Durlauf, hereafter BD (1995, 1996). Their developments utilise unit root-type tests of the Dickey-Fuller (1979) type and consider the existence of stationary differences between pairs of countries. Although these developments are both innovative and appealing, they have generally found little evidence of convergence in the comparisons investigated.

An alternative approach which also is based upon time series data based tests utilises the Kalman filter, time varying parameter approach see for example, St.Aubyn (1996) and Hall, Robertson and Wickens (1992). The Kalman filter may have certain attractions over the BD unit-root testing approach. In particular, it allows the process of catching-up to be identified by viewing the pattern of the time varying time trend. Hall Robertson and Wickens (1992), illustrate this idea via an example. Consider two series which are completely independent of each other and then from a certain point become equal. These series would converge, but would, as we can easily see, fail a stationarity test of the BD type.

When considering tests of convergence, the method of estimation has been important. However, some major criticisms have involve the groups of countries under consideration. One important notion of convergence which has attracted some considerable theoretical interest is the notion of "Convergence Clubs." Via this notion it is assumed that not all countries converge to the same steady state and this is the reason for widespread rejection of convergence. However, conditional upon a set of pre-defining characteristics certain groups may converge. In this paper we will attempt to establish the existence of two such Clubs, a European Club comprising Belgium, France, Germany, Italy, The Netherlands and The United Kingdom and a Nordic Club comprising Denmark, Finland, Sweden and Norway.

2.0 Time series based test of convergence

This type of test generally considers bi-variate differences between two time series of (say) log., (GDP) per capita and studies their properties.

2.1 Unit root-based tests of convergence, Bernard and Durlauf (1996).

Bernard and Durlauf use time-series tests to consider convergence in an explicitly time series setting based upon *differences between countries GDP per capita.* In particular consider the following: define y_i as the log real GDP per capita in country i and likewise y_j for country j. Define the differences in real GDP per capita in countries i and j, $y_i - y_j$.. Define I_t as the information set available at period t. Bernard and Durlauf (1996), p.165, define (Definition 2):
Convergence as equality of long-term forecasts at a fixed time. Countries i and j converge if the long-term forecasts of (log) per capita output for both countries are equal at a fixed time t.

$$\lim_{k\to\infty} E(y_{i,t+k} - y_{j,t+k} | I_t) = 0 \qquad (1)$$

In a time series testing framework, testing such a definition relies upon the time-series properties of the output per capita series. In particular, Bernard and Durlauf (1996), p.170, demonstrate via Proposition 5 that: *If* $y_i - y_j$ *contains either a non zero mean or a unit root, then Definition 2, of convergence is violated.*

2.2 St.Aubyn (1996)

St. Aubyn (1996), defines convergence as follows. Two series , $y_i - y_j$, converge if:

$$(y_i - y_j) \to p\ \varepsilon_t \text{ as } t \to \infty \qquad (2)$$

where $\to p$, means converges in probability and ε_t is a random variable where:

$$E(\varepsilon_t) = D_{XY} \qquad (3)$$

$$\operatorname{var}(\varepsilon_t) = \sigma < 0 \qquad (4)$$

Via (2)-(4), convergence implies that the difference between the two series converges in probability to a third series which is stationary, with constant mean D_{XY} and a constant variance σ. Via (2)-(4) St. Aubyn (1996), relates these characteristics to previous notions of economic convergence, i.e.;

i). Point wise convergence $\to \operatorname{var}(\varepsilon_t) = 0$,
ii). Unconditional convergence $\to D_{XY} = 0$
iii). Conditional convergence $\to D_{XY} \neq 0$.

3.0 The data

The data used relates to real GDP per capita, 1900-1987, and is taken from BD (1995), which is based upon Maddison's (1982, 1989), GDP and population estimates. Figures 1,2 and 3 present the time series plots of the individual series, Figure 1, and the bivariate differences for the candidate "clubs", Figure 2 for Europe and Figure 3 for the Nordic group.

4.0 Kalman filter-based tests of Convergence

The testing procedure and test statistic to be derived is based upon St. Aubyn (1995) and is a version of a convergence test proposed by Hall, Robertson and Wickens (1993).
Define $\Delta_t = (x_i - x_j)_t$, i.e., the difference between the two series under consideration then:

$$\Delta_t = \alpha_t + \varepsilon_t \qquad (5)$$

$$\alpha_t = \alpha_{t-1} + \mu_t \qquad (6)$$

$$\varepsilon_t \sim N(0, \sigma 2) \qquad (7)$$

$$\mu_t \sim N(0, \Omega_t) \qquad (8)$$

$$\Omega_t = \phi 2 \Omega_{t-1} \qquad (9)$$

$$\Omega_0 = \Psi 2 \qquad (10)$$

Written in state-space form equation (5) is the measurement equation and (6) is the state. Equations (5) through (10) have the following properties. The noise variance $\sigma 2$ is assumed to be constant over time although the variance of μ may decline over time. *If* $\phi < 1$ *then* $\phi 2 \to 0$ *in the long run and indicates that that the two series are converging with* $\Delta_t \sim I(0)$, *i.e., stationary.*
The null hypothesis of no convergence implies ϕ=1, such that:
$H_0: \phi = 1;\ H_1: \phi < 1,$.

Table 1
Critical values for $T(\Phi_{ML})$ statistic

Percent	t-value
0.5	-3.702
1	-3.479
5	-2.479
10	-1.970
50	-0.059
99	3.348
99.5	3.529

Taken from St. Aubyn (1996)

St. Aubyn (1996), provides a distribution for the null based upon:

$$T(\phi_{ML}) = \frac{\phi_{ML} - 1}{\sqrt{(h^{-1})_{22}}} \qquad (11)$$

where in (7), $(h^{-1})_{22}$ is the second element in the diagonal of the inverse of the Hessian matrix. Tabulated values taken from St. Aubyn (1996) are reproduced above as Table 1. and will be used in all tests of convergence reported below.

5.0 Results

Table 2a below presents the results for the $T(\phi_{ML})$ statistic. The results treat France as the leader i.e., y_i in all instances where a European Club is considered and likewise Sweden for the Nordic group. For the full sample, 1900-1987, the null hypothesis of non-convergence is rejected in all cases with the weakest result being between France and Germany where rejection is at the 10% level only. On the basis of the full sample period results the concept of a European Convergence Club comprising France, Belgium, Germany, Italy, The Netherlands and the United Kingdom cannot be rejected and likewise a Nordic Club of Sweden, Denmark, Finland and Norway.

Table 2a
Kalman Filter based tests of Convergence

Countries	1900-1987
France-Austria	*-3.359**
France-Belgium	*-2.681**
France-Germany	*-2.297***
France-Italy	*-9.004**
France-Netherlands	*-6.277**
France-UK	*-6.082**
Sweden-Denmark	*-3.143**
Sweden-Finland	*-5.474**
Sweden-Norway	*-2.847**

* denotes significant at the 5% and ** 10% level based upon St.Aubyn (1996)

St.Aubyn finds in his study of convergence where he treats the United States as the leader in all cases that pre- and post- World War Two results differ. Table 2b and 2c below present results for the $T(\phi_{ML})$ statistic for the sample sub-periods 1900-1938 and 1946-1987. The results from Table 2b lead to non-rejection of the non-convergence null for all cases.
The results from Table 2c, however, confirm with somewhat more significant results the full sample conclusions, i.e., rejection of the non-convergence null in all cases, in the case of the European Club, but not non-rejection in the Nordic case.

Table 2b
Kalman Filter based tests of Convergence

Countries	1900-1938
France-Austria	*0.200*
France-Belgium	*1.583*
France-Germany	*1.436*
France-Italy	*-0.154*
France-Netherlands	*0.107*
France-UK	*-0.025*
Sweden-Denmark	*1.339*
Sweden-Finland	*-0.217*
Sweden-Norway	*-0.601*

The sub-sample results seem to imply that, in the European case, convergence occurs most strongly post WWII . However, the full sample convergence implications for the Nordic Club are not supported by either sub-period. This could be due to small sample estimation problems. Alternatively, consider Figure 4 which plots GDP per capita of the four countries in the Nordic group. The shaded area demarcates the pre- post- World War Two period. As can be seen, the series appear to narrow over the full sample, but for the two sub-periods the series appear to look like two sets of parellel lines. This would manifest itself as a non-time varying model over the sub-periods, but over the full sample there appears to be a structural shift caused by WWII. Contrast this with Figure 5 which presents a time series plot of GDP per capita for the European Club members. There a full sample 'narrowing' is observed, an a parellelism in the pre-WWII period. However, very obvious narrowing or convergence is apparent in this later period.

Table 2c
Kalman Filter based tests of Convergence

Countries	1946-1987
France-Austria	*-10.24**
France-Belgium	*-4.336**
France-Germany	*-6.352**
France-Italy	*-2.651**
France-Netherlands	*-4.376**
France-UK	*-3.463**
Sweden-Denmark	*-0.119*
Sweden-Finland	*-1.142*
Sweden-Norway	*1.673*

* denotes significant at the 5% level based upon St.Aubyn (1996)

6.0 Some comparisons with other results.

St.Aubyn (1996), tests for convergence using the Maddison (1979, 1989), data set on GDP per capita using unit root and Kalman filter tests as outlined above. The unit root test results point to convergence between the USA and France, Australia, Belgium, The Netherlands and Switzerland for the period 1890-1989; the USA, France and Germany pre-WWII and the USA, Germany, Italy and Japan, 1947-1989. Via the Kalman filter results he establishes convergence between the USA and Canada, France, Germany, Italy, Japan, UK, Australia, Austria, Belgium, Denmark, Finland, The Netherlands, Norway, Sweden and Switzerland for the period 1890-1989. Non-convergence is not rejected for any countries (relative to the USA), for the pre-war period and for only France, Germany, Italy, Japan and the UK for the period 1947-1989.

Greasley and Oxley (1995, 1997a,b), utilise the BD unit root approach and establish convergence between Australia and the UK, France and Italy; Belgium and The Netherlands and Sweden and Denmark. Bernard and Durlauf (1995), fail to reject the non-convergence hypothesis for any of the countries considered, however, unlike Greasley and Oxley and St.Aubyn, their testing framework does not include the possibility of structural change or sub-sample comparisons which seem to be important.

7.0 Conclusions

In this paper we have utilised the St.Aubyn (1996), Kalman filter based test of convergence utilising the BD data set on GDP per capita, 1900-1987. From this we have been able to establish the existence of a European Convergence Club comprising France, Belgium, Germany, Italy, The Netherlands and the UK where convergence appears to have occurred, in the main, post-WWII. In the case of Sweden, Denmark, Finland and Norway, the results suggest the existence of a Club of four members only if the full sample period is considered. This result may reflect potential problems with any time series-based tests where series track closely in parallel.

8.0 Acknowledgements

We wish to thank Miguel St.Aubyn for providing the GAUSS routines for calculating the $T(\phi_{ML})$ statistic.

9.0 References:

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Figure 1.
GDP per capita (logarithms)

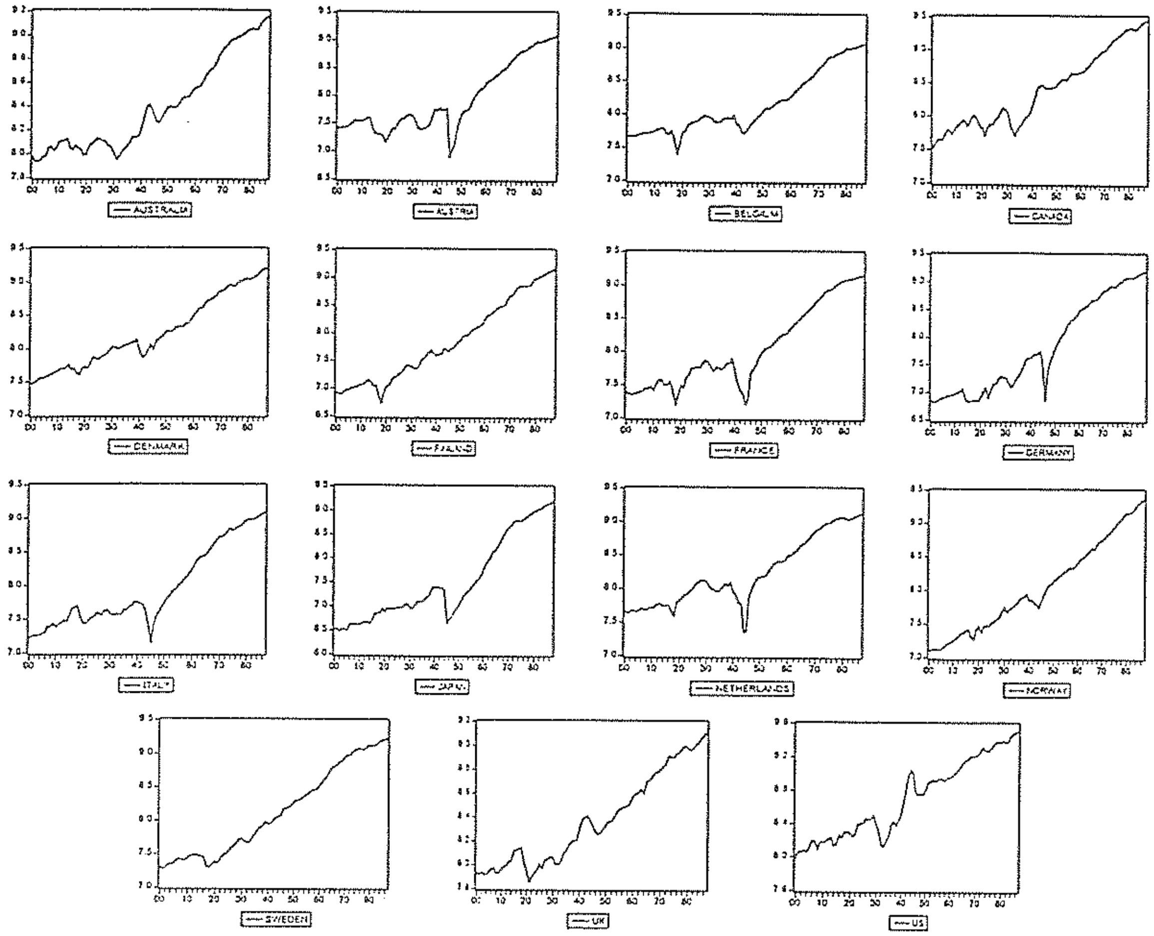

Figure 2
Differences in GDP per capita from France

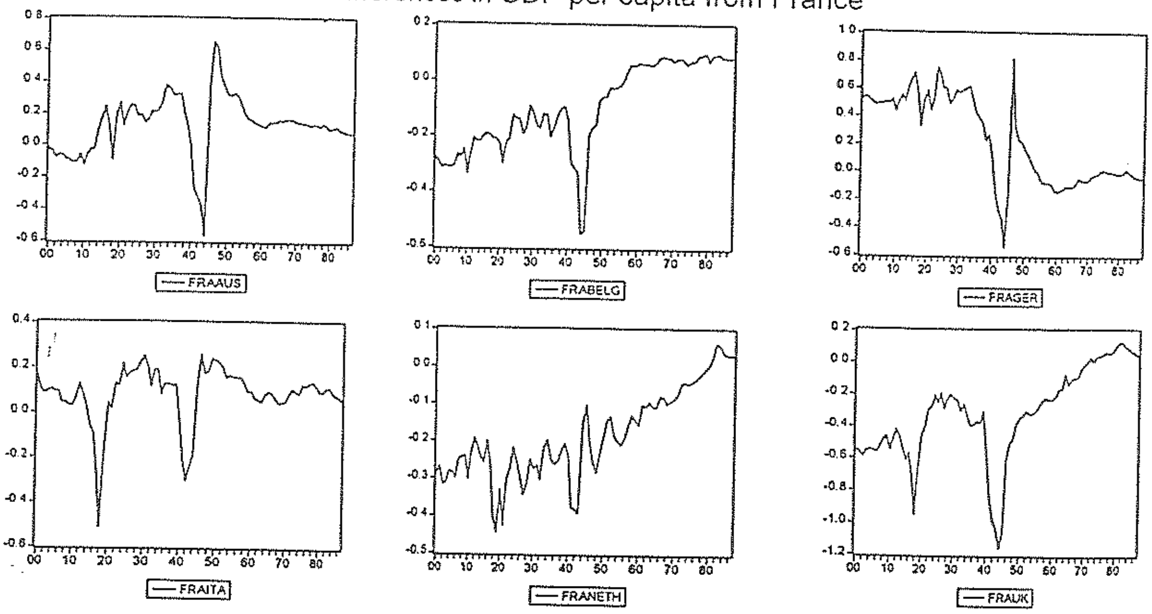

Figure 3
Differences in GDP per capita from Sweden

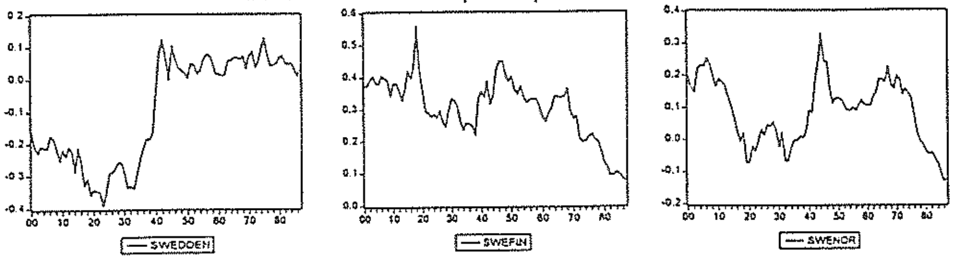

Figure 4
GDP per capita

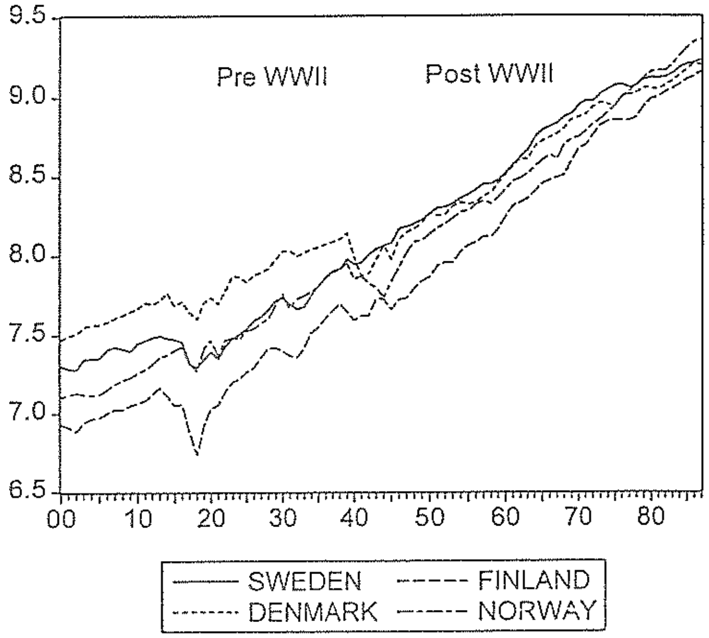

Figure 5
GDP per capita